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Syzygy Tables for the Binary Quintic.

By J. Hammond.

In his Tenth Memoir on Quantics (Phil. Trans., Part II, 1878, pp. 603-661) Prof. Cayley has given an enumeration of the irreducible syzygies for the Binary Quintic, and an incomplete calculation of their values, not extending beyond the deg. order 14.4. Table I of the present paper is meant to replace his enumeration (from which it differs by the omission of twelve syzygies) or that of Prof. Sylvester (*American Journal of Mathematics*, Vol. IV, p. 58). The letters a, b, c, . . . w denote the same system of 23 groundforms as in the memoir referred to, the numbers show the number of irreducible syzygies for any deg. order. The calculated values of all the irreducible syzygies will be found in Table II.

I hope on some future occasion to prove that my enumeration is the correct one, and shall content myself, for the present, with proving that the syzygy named b^5 by Prof. Cayley is reducible.

This syzygy is

$$-8b^5 + a^2q + 10ab^2j - 9abdg + 3ahj + 12b^3h + 32bcm - 12cdj - 4cgh = 0$$

say $\Sigma = 0$.

Copying from Table II the syzygies named d^2, dh, h^2, and dm, we have

$$(d^2) = aj - b^3 + 2bh - cg - 9d^2$$
$$(dh) = am + 2b^2d + cj - 3dh$$
$$(h^2) = adg - bcg - 12bd^2 - 4cm + h^2$$
$$(dm) = ag + 2b^2j - bdg - hj + 12dm.$$

Whence it is easily verified that

$$\Sigma = a(dm) + 8b^2(d^2) - 8b(h^2) - 12d(dh) + 4h(d^2).$$

The remaining eleven syzygies have not been calculated by Prof. Cayley.

It may be noticed that the names of the syzygies in Table II are all of them binary combinations of the groundforms, and it has been found (in every case that I have considered) that every irreducible syzygy contains at least one such binary combination among its terms. But I have no proof of this.

TABLE I.

Order in the Variables.

Degree in the Coefficients.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18
1						*a*												
2			*b*				*c*											
3				*d*		*e*				*f*								
4	*g*				*h*		*i*											
5		*j*		*k*				*l*				1						
6			*m*		*n*		1		1		1		1		1			1
7		*o*				*p*		1		3		1		1		1		
8	*q*		*r*				2		2		3		3				1	
9				*s*		1		5		2		2		2				
10					1		3		4		4				2			
11		*t*				4		3		3		3						
12	*u*				2		4		5				2					
13		*v*		2		3		3		3								
14			1		2		6				3							
15				2		3		4		1								
16					5		2		2		1							
17				2		3		2										
18	*w*		2		2		2		1									
19				2		3												
20			2		1		2											
21				3		1				1								
22			1		2		1											
23		1		1				1										
24			2		1													
25		1				1												
26			2															
27				1														
29		1																
31		1																
36	1																	

Order in the Variables.

Degree in the Coefficients.

TABLE II.

THE SYZYGIES OF THE QUINTIC.

Deg. Order.		Name.
5.11	$ai + bf - ce = 0$	ce
6.6	$aj - b^3 + 2bh - cg - 9d^2 = 0$	d^2
6.8	$ak + 2bi - 3de = 0$	de
6.10	$a^2g - 12abd - 4b^2c - e^2 = 0$	e^2
6.12	$al - 2ci + 3df = 0$	df
6.14	$a^2h - 6acd - 4bc^2 - ef = 0$	ef
6.18	$a^3d - a^2bc + 4c^3 + f^2 = 0$	f^2
7.7	$am + 2b^2d + cj - 3dh = 0$	dh
7.9	$bl + ck - 3di = 0$	di
7.9	$an - b^2e + 2bl + eh = 0$	eh
7.9	$4bl + 2ck - eh + fg = 0$	fg
7.11	$abh - acg + 6bcd + ei = 0$	ei
7.13	$ade - bce + 2cl + fh = 0$	fh
7.15	$a^2bd - ab^2c + ach - 6c^2d - fi = 0$	fi
8.6	$bn - ej + gi + 3dk = 0$	dk
8.6	$ao - 2bn - ej = 0$	ej
8.8	$2b^2h + bcg + 12cm - 3h^2 - ek = 0$	ek
8.8	$adg - bcg - 12bd^2 - 4cm + h^2 = 0$	h^2
8.10	$abk + ap + bde - fj - 6dl = 0$	dl
8.10	$ap + 2cn + fj = 0$	fj
8.10	$bde - cn - hi = 0$	hi
8.12	$3adh + 2bch - 2c^2g - 18cd^2 + el = 0$	el
8.12	$el - 2i^2 + fk = 0$	fk
8.12	$ab^2d - acj + 9cd^2 + i^2 = 0$	i^2
8.16	$3a^2d^2 - abcd - 2b^2c^2 + 2c^2h - fl = 0$	fl
9.5	$aq + 2b^2j - bdg - hj + 12dm = 0$	dm
9.7	$ar + ij + 3dn = 0$	dn
9.7	$ar - b^2k - bp - em = 0$	em
9.7	$co + 6dn + 3em + gl = 0$	gl
9.7	$co + em - hk = 0$	hk
9.7	$bp + co - ij = 0$	ij
9.9	$ab^2g + 6abm - agh + 2bcj - en = 0$	en
9.9	$abm + 3bcj - 3cdg + ik = 0$	ik
9.11	$adk + bck + cp + fm = 0$	fm
9.11	$bck + 2cp + 3d^2e - hl = 0$	hl
9.13	$bei - de^2 - 2il + fn = 0$	fn
9.13	$3abd^2 - b^2cd - 2c^2j + 3cdh + il = 0$	il

TABLE II—Continued.

Deg. Order.		Name.
10.4	$2br - jk + 3do = 0$	do
10.6	$agj + 2b^3g + 12b^2m - 12bdj - 2bgh - eo = 0$	eo
10.6	$2b^2m - 3cq - k^2 - 3hm = 0$	hm
10.6	$4b^2m + 12bdj - 9d^2g + k^2 = 0$	k^2
10.8	$as - 3bdk - 2im - 3dp = 0$	dp
10.8	$2bdk + bgi - deg + 4im + hn = 0$	hn
10.8	$bdk + cr - im = 0$	im
10.8	$2cr - 3dp + jl = 0$	jl
10.10	$bek + in - kl + ep = 0$	ep
10.10	$ep + 2in + fo = 0$	fo
10.10	$abdg + 3adm - ahj + 3cdj - in = 0$	in
10.10	$3adm - 4bcm - 3cdj + kl = 0$	kl
10.14	$bfk - bi^2 + dei + l^2 + fp = 0$	fp
10.14	$9ad^3 + 3bcd^2 + 4c^2m + l^2 = 0$	l^2
11.5	$bs + km - 3dr = 0$	dr
11.5	$b^2o - 6dr - gp - 3jn - 2ho = 0$	ho
11.5	$b^2o + 2bgk - 6bs - 3ho - 3jn = 0$	jn
11.5	$bgk + eq + gp + 2jn + 6km = 0$	km
11.7	$io + kn + er = 0$	er
11.7	$agm - 6bdm + 3bhj - 3dgh + io = 0$	io
11.7	$b^2dg + 12bdm + 2bhj - 3dgh - kn = 0$	kn
11.9	$ajk - bem - 4dej + 2dgi - 2lm + fq = 0$	fq
11.9	$ado + bem + bhk - 2lm + hp = 0$	hp
11.9	$cs + 3d^2k - lm = 0$	lm
11.11	$bik + dek + ip + fr = 0$	fr
11.11	$abdj - b^2cj + 2bcdg + 6cdm - chj + ip = 0$	ip
11.11	$fr - ip - ln = 0$	ln
12.4	$2b^2q + 2bgm + 4bj^2 - 3dgj + ko = 0$	ko
12.4	$bgm + 3hq + ko + 12m^2 = 0$	m^2
12.6	$bdo - 3bjk + 2iq - 2jp + 6ds = 0$	ds
12.6	$bdo + mn + hr = 0$	hr
12.6	$at - bjk + 2mn - jp = 0$	jp
12.6	$bjk - dgk - iq + mn = 0$	mn
12.8	$bk^2 + 2ir + kp - es = 0$	es
12.8	$kp - lo - 2ir = 0$	ir
12.8	$ajm - 2bcq - 2cgm - cj^2 + kp = 0$	kp
12.8	$6bhm - 2cgm - 18d^2m + 3dhj - lo = 0$	lo
12.8	$beo - egk + 3es - 3lo - 3n^2 = 0$	n^2
12.12	$bkl + 2dik + lp + fs = 0$	fs
12.12	$3ad^2j + 2b^2cm + 3bcdj + 2chm + lp = 0$	lp

TABLE II—Continued.

Deg. Order.		Name.
13.3	$2bt - gs + kq + 2jr = 0$	jr
13.3	$bt - jr + mo = 0$	mo
13.5	$bdq + 3bjm - 4dgm - dj^2 - 3au = 0$	au
13.5	$3bdq - 2bjm + 3dgm - kr = 0$	kr
13.5	$3b^2gj - 2bdg^2 + 6bdq + 12bjm - ghj - no = 0$	no
13.7	$bkm - 3d^2o + 2mp - hs = 0$	hs
13.7	$3bkm - ct + 3mp + lq = 0$	lq
13.7	$ct + djk - mp = 0$	mp
13.9	$dk^2 - lr + is = 0$	is
13.9	$3b^2dm + 3bd^2j - 2cjm + 3dhm + lr = 0$	lr
13.9	$ber - bnk + deo - 2lr - np = 0$	np
14.2	$4bgq - 36bu - gj^2 + o^2 = 0$	o^2
14.4	$js + 2mr - 3dt = 0$	dt
14.4	$bgr + 2bjo - gjk + 3nq - 12mr = 0$	mr
14.6	$2b^3q + b^2gm + bhq + bko + et - av = 0$	av
14.6	$bm^2 + d^2q + djm + cu = 0$	cu
14.6	$bko - 2nr + op - et = 0$	et
14.6	$4bm^2 + 9d^2q + 6djm - ks = 0$	ks
14.6	$2bko + gk^2 - 3ks + 3po - 3nr = 0$	nr
14.6	$4bhq + 2bko + 12bm^2 + 6djm - hj^2 + op = 0$	op
14.10	$3akr + bes + 6dkn - 5ekm - 6ls - 6ft = 0$	ft
14.10	$15bd^2m + 4cm^2 + 9d^3j + ls = 0$	ls
14.10	$bkp - 2dkn + ft + p^2 = 0$	p^2
15.3	$bjq - 6dgq + gjm + 2j^3 + or + 54du = 0$	du
15.3	$2bjq - bv + gjm - or = 0$	or
15.5	$bmo - dgr - djo - gkm - 3eu = 0$	eu
15.5	$2b^2t - 2dgr - 2djo - pq - ht = 0$	ht
15.5	$bmo + 3djo + 2gkm + 3ht - 6ms = 0$	ms
15.7	$6dhq + 3dko + 18dm^2 + 3hjm + 2cjq - cv = 0$	cv
15.7	$dko - pr + it = 0$	it
15.7	$bkr + it + pr + ns = 0$	ns
15.7	$5b^2jm + 3bdj^2 - jk^2 - 3cv + 3pr = 0$	pr
15.9	$adt + 3amr - 2diq + 6dmn - 3em^2 - 3fu = 0$	fu

TABLE II—Continued.

Deg. Order.		Name.
16.4	$3djq + 2j^2m + 2r^2 - 3dv = 0$	dv
16.4	$6b^2u + 3bmq + djq - 2dv - 3hu = 0$	hu
16.4	$6b^2u + 2bmq - 3dv + kt = 0$	kt
16.4	$kt - 2r^2 - os = 0$	os
16.4	$3b^2u + 2bmq + gm^2 - r^2 = 0$	r^2
16.6	$agt - 4bjs - 20bmr + 2bnq - ev = 0$	ev
16.6	$bdt + bmr + dkq + jkm + 3iu = 0$	iu
16.8	$9adu + 2b^2m^2 - 3bd^2q + 2cmq + 2k^2m - lt = 0$	lt
16.8	$2dkr - lt + ps = 0$	ps
16.10	$aht - 6bdkm - 18dmp + 2fjq - 2hiq - fv = 0$	fv
17.3	$boq + gjr + gmo - 2gkq + 2j^2o + 18ku = 0$	ku
17.3	$3ku - qs - 2mt = 0$	mt
17.5	$2b^2jq - b^2v - 2bdgq + 18bdu + nt + hv = 0$	hv
17.5	$bor - 2gkr - jko + 6rs + 3nt = 0$	nt
17.5	$9bdu + 3dmq + 2jm^2 - rs = 0$	rs
17.7	$2ajt - 18d^2t - 18dmr + 6dnq + ijq - iv = 0$	iv
17.7	$d^2t + dmr - km^2 + lu = 0$	lu
18.2	$4gmq + 2j^2q - 36mu - ot - jv = 0$	jv
18.2	$3bgu - bq^2 - jv + ot = 0$	ot
18.4	$4bjt - 2bqr - 3dgt + kv = 0$	kv
18.4	$dgt + doq + 2gmr + 2jmo - 6nu = 0$	nu
18.6	$2bos + 3br^2 - 3dor + gks - 3s^2 + 3pt = 0$	pt
18.6	$27d^2u - 4m^3 - s^2 = 0$	s^2
18.8	$bdkq + 9deu - 4imq + 6m^2n - lv = 0$	lv
19.3	$3bju - rt + mv = 0$	mv
19.3	$6bju - 9dgu + 3dq^2 + 2jmq + 2rt = 0$	rt
19.5	$2j^2s + 10jmr - jnq - nv + 6aw = 0$	aw
19.5	$3bmt + djt + 2dqr + 3pu = 0$	pu
19.5	$b^2gt + b^2qo + 12bmt - ght - hoq - jnq - nv = 0$	nv
20.2	$6bw + gjt - 2gqr - joq + 18ru = 0$	ru
20.2	$12bw + gjt - ov = 0$	ov
20.4	$6bmu + 9dju + 2m^2q - st = 0$	st
20.6	$3bjmo + 5dj^2o + 2dgjr + 2jpq - pv - 12cw = 0$	cw
20.6	$5bjmo + 3dj^2o - 9giu + 3iq^2 - 2kor + 3pv = 0$	pv

TABLE II—Continued.

Deg. Order.		Name.
21.3	$2j^2t - jqr + rv + 18dw = 0$	dw
21.3	$3bou - 2bqt - gmt + rv = 0$	rv
21.3	$4bou - 12dw - gku - gqs + 2moq + 12su = 0$	su
21.5	$gnt - 2jos - 10mor + noq - 6ew = 0$	ew
21.9	$bioq + 2crt - deoq - 6dkmo - 6dpt - hnt + 6fw = 0$	fw
22.2	$3gmu + 3j^2u - mq^2 - t^2 = 0$	t^2
22.4	$3dgoq - 27dou + g^2mr + 2gjmo - 6jmt - o^2r - 3mqr - 9hw = 0$	hw
22.4	$6bru + 3dqt + 2jmt - sv = 0$	sv
22.6	$3bmo^2 + 5djo^2 + 2dgor - gpt + 2poq - 12iw = 0$	iw
23.1	$3gou - 4gqt - oq^2 + 36tu - 12jw = 0$	jw
23.3	$grt + 2jot - oqr + 6kw = 0$	kw
23.7	$bmor + j^2kr + 2jpt - rht + 6lw = 0$	lw
24.2	$3jou - 2jqt + vt + 12mw = 0$	mw
24.2	$3gru + 3jou - q^2r - tv = 0$	tv
24.4	$g^2r^2 + 2gjor + j^2o^2 - 6jrt - 3qr^2 + 9nw = 0$	nw
25.1	$3g^2ju - gjq^2 - 4gqv + 36uv - 12ow = 0$	ow
25.5	$bgrt + bjot - gmor - jmo^2 - jst + mrt + 3nou - 3pw = 0$	pw
26.2	$gt^2 + 3uo^2 - 2qot + 12rw = 0$	rw
26.2	$3gj^2u - 2jqv + 12rw + v^2 = 0$	v^2
27.3	$2jt^2 - 3rou + qrt + 6sw = 0$	sw
29.1	$6gjqu + 3guv - 108ju^2 + 2jq^3 - q^2v + 12tw = 0$	tw
31.1	$3g^2tu - gq^2t + 6goqu + 2q^3o - 108u^2o + 12vw = 0$	vw
36.0	$g^3u^2 + 2g^2q^2u + gq^4 - 72gqu^2 - 8q^3u + 432u^3 - 16w^2 = 0$	w^2